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Analysis Procedures for Al Activation Studies

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Abstract

Using a HPGe detector at the Fermilab Radiation Analysis Facility, we measure the 1274 keV γ ray from activation of ^{22}Na induced in Al tags in order to provide a measure of the hadron flux in the collimation region of the Main Injector. Data and some procedures will be documented.

1 Introduction

The activation of ^{22}Na in an ^{27}Al target by a flux of secondary hadrons is represented adequately by a constant cross section of 10.1 millibarns per nucleus above a threshold of 30 MeV. For most applications, the half life is sufficiently long (2.60 years) that a measurement of the induced activation can be converted to a fluence estimate (integrated hadron flux). We will document that procedure along with a more accurate flux estimate which uses the history of the activation flux.

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2 Formulas

2.1 Activity and Activation

For a sample with N_I atoms of isotope I which has a half life of $t_{1/2}$ (or mean lifetime of τ), the number of atoms changes by decay in accordance with

$$\mathcal{N}(t) = N_I\, e^{-\frac{t}{\tau}} = N_I\, 2^{-\frac{t}{t_{1/2}}} \tag{1}$$

The number of decays, S, also known as the activity of the sample, is thereby

$$S(t) = -\frac{d\mathcal{N}}{dt} = \frac{N_I}{\tau} e^{-\frac{t}{\tau}} \tag{2}$$

So the initial activity is

$$S = \frac{N_I}{\tau} = \frac{N_I \ln 2}{t_{1/2}} \tag{3}$$

This is the activity in becquerel (Bq). To get the activity in Curies, one divides by 3.7×10^{10}. For the activity in pico Curies (pCi) one divides by 3.7×10^{-2}.

To obtain the specific activity, S_A, one expresses the number of atoms in moles (m) or grams (M). For a material with atomic mass A, the number of moles is $m = \frac{M}{A}$.

$$S_A(Bq/mole) = \frac{S}{m} = \frac{N_A \ln 2}{t_{1/2}} \tag{4}$$

$$S_A(Bq/gm) = \frac{S}{M} = \frac{N_A \ln 2}{A\, t_{1/2}} \tag{5}$$

$$S_A(pCi/gm) = \frac{S}{3.7 \times 10^{-2} M} = \frac{N_A \ln 2}{3.7 \times 10^{-2} A\, t_{1/2}} \tag{6}$$

where N_A is the Avogadro Constant.

2.2 Cross Sections and Fluence

In a beam of particles, nuclear interactions produce new isotopes. The number of new nuclei is proportional to the fluence, Φ, measured in particles per unit area (particles-cm^{-2}); the rate of production is proportional to the flux, $\frac{d\Phi}{dt}$ (particles-cm^{-2}sec^{-1}). In a material with n_T target atoms per unit volume, an interaction with cross section σ will produce n_I atoms per unit volume of isotope I

$$n_I = \Phi n_T \sigma. \tag{7}$$

The activity, S_A (Bq per cm^3), produced by n_I atoms per cm^3

$$S_A = \frac{n_I \ln 2}{t_{1/2}} = \frac{\Phi n_T \sigma \ln 2}{t_{1/2}} \tag{8}$$

We will want the specific activity per gram of target material, $S_A = S/\rho_T$ (Bq per gram).

$$S_A(Bq/gm) = \frac{n_I \ln 2}{\rho_T\, t_{1/2}} = \frac{\Phi n_T \sigma \ln 2}{\rho_T\, t_{1/2}} \tag{9}$$

Substituting for n_T with $\rho_T N_A / A_T$ we have

$$S_A(Bq/gm) = \frac{\Phi N_A \sigma \ln 2}{A_T t_{1/2}} \tag{10}$$

$$S_A(pCi/gm) = \frac{\Phi N_A \sigma \ln 2}{A_T t_{1/2} 3.7 \times 10^{-2}} \tag{11}$$

We calculate the fluence from the Specific Activity as

$$\Phi = \frac{S_A A_T t_{1/2} 3.7 \times 10^{-2}}{N_A \sigma \ln 2} \tag{12}$$

2.3 Activation of ^{27}Al

The reaction ^{27}Al $\rightarrow$ ^{22}Na has a total cross section, $\sigma = 10.1 \times 10^{-27}$ cm^2 per atom with a threshold at about 30 MeV (see Fig. IV.28 on Barbier[1], Page 194). Taking $N_A = 6.022 \times 10^{23}$, $A_T = 26.98$, $t_{1/2} = 2.6027$ years or 82.135×10^6 seconds we find

$$\Phi(hadrons/cm^2) = 1.9448 \times 10^{10} S_A(pCi/gm) \tag{13}$$

2.3.1 Particle Flux with Decay Correction from Exposure History

For a hadron flux produced by 8 GeV proton beam losses, we assume that the spatial distribution of shower particles remains relatively fixed with time variations being due to beam quality (halo) and/or program requirement changes[2]. This implies that the fluence of hadrons, Φ, at a sampling point near a secondary collimator, for example, will be proportional to the signal integral on a nearby loss monitor, L.

$$\Phi = \varepsilon L \tag{14}$$

When we relate activity of ^{22}Na to the fluence we will correct for decays. For Main Injector operation, loss monitor sums, L_i, are recorded for each acceleration cycle. As developed in [2], tools are available for adding these to provided loss history in either 10 minute or 1 week intervals. Weekly sums are sufficient for correcting for ^{22}Na decay (2.6027 year half life).

$$LI_j = \sum_{t=t_j}^{t_j+T_s} LI(t) \tag{15}$$

To account for decays, we will weight these to provide an exponentially weighted sum but express the life time using the half life

$$LW(I, T_M) = \sum_j LI_j 2^{-(T_M-T_j)/t_{1/2}} \tag{16}$$

where T_M is the radiation measurement time, T_j is the quanta time and $t_{1/2}$ is the half life for isotope I. With times in seconds, LW is in units of Rads. Note that this is not the weighted quantity used in Reference[2], since we do not scale by the inverse half life.

We will also want the sum loss without weighting

$$L(I, T_M) = \sum_j LI_j \tag{17}$$

We can provide the fluence (corrected for decays) by correcting the uncorrected fluence in Eq. 13 using

$$\Phi = \Phi_{uncorr} \frac{L(I,T_M)}{LW(I,T_M)} \tag{18}$$

2.3.2 Particle Fluence for Uniform Irradiation

To compare with some typical formulas for activation analysis, we will consider the case where the fluence is delivered in a flux which is uniform in time. For $d\Phi/dt$ a constant for irradiation time from 0 to t_i (from Barbier[1], p 15). This will produce n_I nuclei of isotope I per unit volume

$$n_I(t) = n_T \sigma \frac{d\Phi}{dt} \int_0^{t_i} e^{-(t_i-\tau)/\tau_I} d\tau \tag{19}$$

$$n_I(t) = n_T \sigma \frac{d\Phi}{dt} \tau_I (1 - e^{-t_i/\tau_I}) \tag{20}$$

After a cooling time, t_c, the number of atoms will have decayed to

$$n_I(t_c) = n_T \sigma \frac{d\Phi}{dt} \tau_I (1 - e^{-t_i/\tau_I}) e^{-t_c/\tau_I} \tag{21}$$

So if $n_I(t_c)$ atoms per cm^3 of isotope, I, remain after a uniform irradiation for t_i and cool down t_c, we will have activity of $S_A(Bq/gm) = n_I/(\tau_I \rho_T)$. Again we can substitute for n_T

$$S_A(t_c)(Bq/gm) = \frac{N_A}{A_T} \sigma \frac{d\Phi}{dt} (1 - e^{-t_i/\tau_I}) e^{-t_c/\tau_I} \tag{22}$$

alternatively, using $M/A_T = N_T/N_A$ we have

$$S_A(t_c)(Bq/gm) = \frac{N_T}{M} \sigma \frac{d\Phi}{dt} (1 - e^{-t_i/\tau_I}) e^{-t_c/\tau_I} \tag{23}$$

Now, using pCi rather than Bq, we calculate the flux

$$\frac{d\Phi}{dt} = \frac{M\,3.7 \times 10^{-2}}{N_T \sigma} \frac{e^{t_c/\tau_I}}{(1 - e^{-t_i/\tau_I})} S_A(t_c)(pCi/gm) = \frac{A_T\,3.7 \times 10^{-2}}{N_A \sigma} \frac{e^{t_c/\tau_I}}{(1 - e^{-t_i/\tau_I})} S_A(t_c)(pCi/gm) \tag{24}$$

We convert this corrected flux to a fluence by multiplying by the irradiation time, t_i.

$$\Phi(hadrons/cm^2) = \frac{d\Phi}{dt} t_i = \frac{A_T\,t_i\,3.7 \times 10^{-2}}{N_A \sigma} \frac{e^{t_c/\tau_I}}{(1 - e^{-t_i/\tau_I})} S_A(t_c)(pCi/gm) \tag{25}$$

For the uniform exposure assumption we find

$$\Phi = \Phi_{uncorr} \frac{t_i}{t_{1/2} ln2} \frac{e^{t_c/\tau_I}}{(1 - e^{-t_i/\tau_I})} \tag{26}$$

2.3.3   Check Uniform Irradiation Against Irradiation History

For long half life isotopes, the correction for irradiation exposure in the Main Injector is relatively easy since the flux relation between BLM locations and residual radiation locations has remained sufficiently constant at most locations. For ^{22}Na with its 2.6027 year half life, corrections using weekly exposure sums are adequate. To obtain fluence corrected for decays at the locations used for Al Tags, we identify a nearby loss monitor and use Eq 18 for correction. In Table 1 we show the correction for uniform exposure assumption along with the exposure-weighted results for our six BLM's. Al Tags were installed on 10/12/2007.

Table 1: Comparison of Fluence Exposure Corrections

BLM	End Irradiation	Unweighted Dose (Rads)	Weighted Dose (Rads)	Weighted Irradiation Correction	Uniform Irradiation Correction
LI230	10/08/2008	1014685.056	926161	1.095	1.138
	08/26/2009	3007399.450	2502857	1.201	1.270
	08/12/2010	3511384.733	2382854	1.473	1.424
LI301	10/08/2008	280598.774	256202	1.095	1.138
	08/26/2009	565883.539	454461	1.245	1.270
	08/12/2010	1424778.163	1121736	1.270	1.424
LI302	10/08/2008	796000.000	725642	1.096	1.138
	08/26/2009	1560000.000	1248042	1.249	1.270
	08/12/2010	3610000.000	2830470	1.275	1.424
LI303	10/08/2008	1370000.000	1251607	1.094	1.138
	08/26/2009	2570000.000	2042957	1.257	1.270
	08/12/2010	4840000.000	3596827	1.345	1.424
LI307	10/08/2008	415700.000	390319	1.065	1.138
	08/26/2009	1199000.000	992500	1.208	1.270
	08/12/2010	1958000.000	1429000	1.370	1.424
LI309	10/08/2008	415700.000	390319	1.065	1.138
	08/26/2009	1199000.000	992500	1.208	1.270
	08/12/2010	1958000.000	1429000	1.370	1.424

Note that we seek the fluence produced by a proton loss. The to get the Activation/BLM with correction one increases the Activation by the correction or decreases the unweighted BLM reading. The uniform irradiation correction is typically a few percent larger than the weighted correction. This is because the collimation system commissioning during this period saw increased losses later in the exposure. The exception is for LI230 for the longer period since the tuning eventually improved the loss at the primary collimator.

A step in understanding the precision of the activation analysis can be obtained by comparing the activation (pCi/gm) with the BLM loss (Rads) with decay correction for two successive measurements. Table 2 shows the ratio S_A/L and S_A/LW for collimator, quadrupole and some of the mask locations. Unfortunately the installation of samples secured to the side of the concrete mask material by duct tape failed during the first or second year, yielding no data for comparison at those locations.

We include the data for the tag at C308 despite the addition of masks between C307 and C308 which modified the flux at the C308 tag location. We see that with the exception of the C308 data,

Table 2: Comparison of One Year and Two Year Activation per Loss

Tag Location	BLM	$\frac{S_A}{L}$	$\frac{S_A}{LW}$
		2009/2008	2009/2008
C301	LI302	0.963	1.097
C303	LI303	0.947	1.088
C307	LI307	1.043	1.183
C308	LI309	1.273	1.456
STMM303	LI303	0.735	0.845
STMM308	LI309	0.876	1.002
Q301DS	LI301	0.836	0.951
Q303DS	LI303	0.851	0.970
Q307DS	LI307	0.992	1.126
Q230DS	LI230	1.016	1.114
Q302US	LI302	0.810	0.922

the ratio of two year divided by one year data shows agreement to 15% when corrected for decay. Even without the decay correction, the two year measurement agrees with the one year measurement to better than 25%.

Using the Al Tags activation to determine the fluence gives a ratio of fluence at the tags to BLM signal at a nearby loss monitor. Calibration of the BLM to the lost beam can be done based on either sums from weekly or longer times or on measurements on one or a few pulses. For this note we will calibrate to the distribution of uncaptured beam loss on one pulse. Using the uncaptured beam loss and the signals of all the BLM's in the collimation region, we get a crude loss measurement. Comparisons with other loss determinations will be done later. MARS results for the fluence can be compared with these measurements based on the proton loss in each secondary collimator.

Using a single pulse of $23 Cycle Beam (PBar plus NuMI beam), a measurement of the collimator loss was used for a crude calibration of the beam loss monitor response. The uncaptured beam loss from a pulse in which 34.83E12 protons was injected was 1.12E12 (3.2%). This was allocated to the 4 secondary collimators in accordance with the sum of BLM signal in the two adjacent loss monitors (LI301 + LI302 for C301); (LI303 + LI304 for C303); (LI307 + LI308 for C307); (LI309 for C308). It is then assumed that either of the pair of loss monitors can be used to determine the protons lost on that collimator. Applying this to LI308 would be OK for a fixed geometry but masks were added twice since the initial collimator installation so we will avoid further use of LI308 for this study. Data was recorded in file pfl-11June13-143510-amc01.csv and analyzed in same file name with extension .xlxs. Table 3 shows the calibration values used.

Using these calibration values, we now provide the Fluence at the Al Tag locations normalized to the lost protons inferred from nearby loss monitors. Table 4 shows the data from this activation study. We include the tag and BLM locations, activation, BLM sum, fluence, lost protons and fluence per lost proton.

Table 3: Calibration of BLM readings

BLM	Rads/E12Protons
LI301	0.114654036
LI302	0.331613821
LI303	0.361673392
LI304	0.084594465
LI307	0.242013954
LI308(skip)	0.204253903
LI309	0.446267857

3   Appendix A - List of Symbols

A Atomic Mass Number or Atomic Weight

N_A Avogadro Constant (Avogadro's number), $6.02214179(30) \times 10^{23}$ mol-1 (gram).

S_A Specific activity (in Bq) – number of decays per second per amount of substance

4   Acknowledgments

This note is intended only to document this effort. The contributions from Vernon Cupps, Gary Lauten and others are the basis for this report.

References

[1] Marcel Barbier. *Induced Radioactivity*. North-Holland Publishing Company, Amsterdam, London, 1969.

[2] Bruce C. Brown and Guan Hong Wu. Measuring Correlations Between Beam Loss and Residual Radiation in the Fermilab Main Injector. In Jan Chrin, editor, *Proceedings of the 46th ICFA Advanced Beam Dynamics Workshop on High-Intensity and High-Brightness Hadron Beams (HB2010)*, Morschach, Switzerland, 2010. Also available as FERMILAB-CONF-10-368-AD.

Table 4: Calibration of BLM readings. Tags installed 10/12/07
* indicates tag not found at installation position.

Location	Tag ID	Date Rem	Activ	BLM	UnWt BLM	fluence	Lost	Fluence per
			pCi/gm		Rads	hadrons/cm^2	Protons	Proton Lost
C301	6510	10/08/08	230	LI302	7.96E+005	4.907E+12	2.40E+018	2.04E-006
	6198	08/26/09	434		1.56E+006	8.440E+12	4.70E+018	1.79E-006
	6187	08/12/10			3.61E+006			
C303	6368	10/08/08	777	LI303	1.3700E+06	1.511E+13	3.7879E+18	3.99E-006
	6503	08/26/09	1380		2.5700E+06	2.683E+13	7.1059E+18	3.78E-006
	6548	08/12/10			4.8400E+06			
C307	6325	10/08/08	339	LI307	4.1570E+05	6.593E+12	1.7177E+18	3.84E-006
	6168	08/26/09	1020		1.1990E+06	1.984E+13	4.9543E+18	4.00E-006
	6559	08/12/10			1.9580E+06			
C308	6318	10/08/08	1690	LI309	5.6961E+05	3.287E+13	1.2764E+18	2.58E-005
	6833	08/26/09	5210		1.3792E+06	1.013E+14	3.0905E+18	3.28E-005
	6534	08/12/10			2.2711E+06			
STCM301	6820*	10/08/08	2310	LI302	7.9600E+05	4.493E+13	2.40E+018	1.87E-005
	6834*	08/26/09	14300		1.5600E+06	2.781E+14	4.70E+018	5.91E-005
	6507*	08/12/10			3.6100E+06			
STCM303	6842	10/08/08	7430	LI303	1.3700E+06	1.445E+14	3.7879E+18	3.81E-005
	6167*	08/26/09	6660		2.5700E+06	1.295E+14	7.1059E+18	1.82E-005
	6811*	08/12/10			4.8400E+06			
STCM308	6828*	10/08/08	1210	LI309	5.6961E+05	2.353E+13	1.2764E+18	1.84E-005
	6822*	08/26/09	10300		1.3792E+06	2.003E+14	3.0905E+18	6.48E-005
	6319*	08/12/10			2.2711E+06			
STMM301	6841	10/08/08	5440	LI302	7.9600E+05	1.058E+14	2.40E+018	4.41E-005
	6821*	08/26/09	9170		1.5600E+06	1.783E+14	4.70E+018	3.79E-005
	6110*	08/12/10			3.6100E+06			
	6518*	08/12/10						
STMM303	6502	10/08/08	6690	LI303	1.3700E+06	1.301E+14	3.7879E+18	3.43E-005
	6538	08/26/09	9230		2.5700E+06	1.795E+14	7.1059E+18	2.53E-005
	6659	08/12/10			4.8400E+06			
STMM308	6169	10/08/08	3070	LI309	5.6961E+05	5.971E+13	1.2764E+18	4.68E-005
	6335	08/26/09	6510		1.3792E+06	1.266E+14	3.0905E+18	4.10E-005
	6523	08/12/10			2.2711E+06			
Q301DS	6195	10/08/08	3510	LI301	2.8060E+05	6.826E+13	2.4474E+18	2.79E-005
	6605	08/26/09	5920		5.6588E+05	1.151E+14	4.9356E+18	2.33E-005
	6200	08/12/10			1.4248E+06			
Q303DS	6831	10/08/08	1960	LI303	7.9600E+05	3.812E+13	2.2009E+18	1.73E-005
	6330	08/26/09	3270		1.5600E+06	6.360E+13	4.3133E+18	1.47E-005
	6511	08/12/10			3.6100E+06			
Q307DS	6516	10/08/08	573	LI307	4.1570E+05	1.114E+13	1.7177E+18	6.49E-006
	6515	08/26/09	1640		1.1990E+06	3.190E+13	4.9543E+18	6.44E-006
	6344	08/12/10			1.9580E+06			
Q230DS	6327	10/08/08	2570	LI230	1.0147E+06	4.998E+13		
	6513	08/26/09	7740		3.0074E+06	1.505E+14		
	6521	08/12/10			3.5114E+06			
Q302US	6275	10/08/2008	8320	LI302	7.96E+005	1.618E+14	2.40E+018	6.74E-005
	6504	08/26/2009	13200		1.56E+006	2.56719E+14	4.70E+018	5.46E-005
	6508	08/12/2010			3.61E+006			